LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2025**

PST2MC01 – ESTIMATION THEORY

Date: 24-04-2025　　Dept. No. ☐　　Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION A – K1 (CO1)

Answer ALL the questions **(5 x 1 = 5)**

1 Define the following

a) Unbiased estimator.

b) Sufficient statistic.

c) Completeness of family of distributions.

d) Consistent estimator.

e) Bayes' estimator.

SECTION A – K2 (CO1)

Answer ALL the questions **(5 x 1 = 5)**

2 True or False

a) Let X_1, X_2 be a random sample of size 2 from B(1, θ) , $0 < \theta < 1$. Then $X_1 + X_2$ is sufficient for θ.

b) The binomial distribution does not belong to exponential family.

c) Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from P(θ) , $\theta > 0$. Then sample mean is M.L.E. of θ.

d) The Lehmann-Scheffe theorem provides UMVUE.

e) Jackknife is not a re-sampling method.

SECTION B – K3 (CO2)

Answer any THREE of the following **(3 x 10 = 30)**

3 Establish the properties of class of all unbiased estimators of g(θ).

4 Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from U[0, θ] , $\theta > 0$. Show that the n^{th} order statistic is sufficient for θ.

5 Show that the family of normal distributions is complete.

6 Let the following table

x	0	1	2	3	4	5
f	5	11	15	12	6	1

represent a summary of a sample of size 50 from a binomial distribution having n = 5. Find the maximum likelihood estimate of $P(X\geq4)$.

7 Explain Jackknife and Bootstrap methods.

SECTION C – K4 (CO3)	
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Suppose $X_1,X_2,\ldots,X_n$ is a random sample from the distribution with the p.d.f. $f(x;\theta) = \theta\exp(-\theta x)$, $\theta > 0$, $x > 0$, find MVBE of $1/\theta$.
9	(a) Show that M.L.E. is not unique and not sufficient with an example each. (b) Prove the invariance property of CAN estimator. (6.5+6)
10	Let $X_1,X_2,\ldots,X_n$ be a random sample from $N(\theta,1)$. Show that sample mean and sample variance are independent with the help of Basu's theorem.
11	Let $X \sim (0, \theta)$, $\theta>0$. Assume that the prior distribution of θ is $h(\theta) = \theta exp\ (-\theta)$, $\theta>0$. Find Bayes' estimator of θ if the loss function is (i) squared error and (ii) absolute error. (7.5+5)
SECTION D – K5 (CO4)	
	Answer any ONE of the following **(1 x 15 = 15)**
12	State and prove the Cramer-Rao inequality after stating the regularity conditions.
13	(a) If δ_1^* is UMVUE and δ_2^* is bounded UMVUE then show that $\delta_1^* . \delta_2^*$ is also UMVUE. (9) (b) State and prove the Rao-Blackwell theorem. (6)
SECTION E – K6 (CO5)	
	Answer any ONE of the following **(1 x 20 = 20)**
14	(a) Let $X_1,X_2,\ldots,X_n$ be i.i.d. $P(\theta)$, $\theta > 0$. Show that sample mean is UMVUE of θ. (10) (b) Show with an example that an estimator is consistent but not consistent asymptotically normal. (10)
15	Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from $N(\mu,\sigma^2)$, $\mu \in R$ & $\sigma^2 > 0$. Find M.L.E. of (i) μ when σ^2 is known (ii) σ^2 when μ is known and (iii) $\theta = (\mu,\sigma^2)$ when μ and σ^2 are unknown.

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